Functions, Inverse Functions and the Modulus Function

Definitions

Function

A rule that changes a single number to another <u>single</u> number is a function. (Also known as one to one, or many to one mapping).

Domain

The domain of a function is the set of numbers that the rule operates on. (often the allowable x or independent variable values).

Codomain

The codomain of a function is the set of values that the function **could** (but might not) produce as output.

Range

The range of a function is the set of values that the function **does** produce. (as output). This is always a subset of the codomain.

Function of a Function

The output of the first function is the input to the second function. (eg f(g(x)) written fg – note that g operates first)

Inverse Functions

The inverse function (if it exists) 'undoes' the original function. $f^{-1}(f(x)) = x$; $f(f^{-1}(x)) = x$.

Modulus Functions

Modulus function written $|f(x)|$ is defined as $f(x)$ if $f(x) \geq 0$ and $-f(x)$ if $f(x) < 0$. ie $|f(x)|$ is always >0.

Note that this <u>different from</u> $f(|x|)$ which = $f(x)$ if $x \geq 0$ and = $f(-x)$ if $x < 0$. $f(|x|)$ can be negative.

Even and Odd Functions

$f(x)$ is an even function if $f(x) = f(-x)$ and an

odd function if $f(x) = -f(-x)$.

Manipulation and Tips

The inverse function can only exist for a one to one function. To find the inverse function transpose x for y and re-arrange for y.

The range of the inverse function is the domain of the function, and the domain of the inverse function is the range of the function.

Examples of inverse functions

- Inverse of sin(x) is $\sin^{-1}(x)$
- Inverse of $\log_e x$ is e^x

Injective, Surjective and Bijective Functions

The easy one is bijective. A function is said to be **bijective** if it is both **injective** and **surjective.**

Injective - a function is injective if there is a unique output for each input (one to one mapping). Injective functions can have inverses.

Surjective – a function is surjective if the range is the same as the codomain.

Notes

(i) Functions strictly apply to elements of a set. In the above, for simplicity, 'numbers' are used for set 'elements'.

(ii) Some notation in general use

- f(x) — the value of the function f at x
- $f: A \rightarrow B$ — f is a function under which each element of set A has an image in set B where set A is the Domain, set B the Codomain
- $f: x \mapsto y$ — the function f maps the element x to the element y
- $f \circ g$ or fg — the composite function (or function of a function) where $f \circ g\,(x) = fg(x) = f(g(x))$